Expansion and Contraction of an Air-Water Mixture in Vertical Flow

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An experimental two-phase flow study was made on an air-water system at atmospheric pressure to obtain information on the effect of expansion and contraction of flow area on the relative velocity of the two phases. The data show that the relative velocity and hence the mean void fraction of the air-water mixture changed following either an expansion or contraction; however the magnitude of the change was not great and could be predicted by a semitheoretical equation. The air-water data are also compared with data taken from a steam-water system at 150 to 600 lb./sq. in. In addition, a photographic study was made of the transition zone, and phase distributions were obtained by the use of a radiation attenuation traversing technique.

Prediction of the density of an adiabatic two-phase fluid in motion is difficult because of inadequate information concerning the relative velocity between the gas and liquid phases. The gas-volume fraction is interrelated with the velocity of the two phases and the gas-weight fraction through the continuity equation. At the present time all the factors that affect the relative velocity, or "slippage," between phases are not well established. There is meager experimental evidence which indicates that the relative velocity of the two phases is a function of the liquid flow rate, the gas-weight fraction, pressure, and possibly the geometry of the flow path.

Behringer (*1*) in a series of experiments on a static steam-water system showed that the relative velocity of the two phases decreases with increasing pressure. Marchaterre (*2*) observed a similar effect over a pressure range of 25 to 600 lb./sq. in. with a natural circulation system. He also noted an effect of liquid flow rate on the relative velocity of the two phases. Schurig (*3*) reported a marked influence of the circulation rate on the relative velocity. The recent data of Lottes *et al.* (*4*) for a natural- and a forced-circulation boiling system shows both a flow rate and a quality effect on the slippage between the two phases. They obtained local two-phase density measurements by gamma-ray attenuation methods and readily converted the data to a slip ratio V_g/V_w. Cook (*5*), on the basis of an extensive two-phase density study of steam water in multisection rectangular channels, found that an increase of V_g/V_w occurs with length along the heated channel, the increase is a function of the rate of vaporization, and with no vaporization V_g/V_w is merely a function of geometry and the volume flow of the two phases. Dengler (*6*), using a radioactive-tracer technique, measured liquid holdups in a 1-in.-diameter pipe and correlated them as a function of the mixture quality. Eddy (*7*) obtained local density values at atmospheric pressure for steam-water mixtures in a horizontal tube and showed the distribution of each phase in the tube. Sher (*8*) performed similar tests in a vertical tube. His vapor volume fraction-quality data were approximately 10% higher than the data of Martinelli, *et al.*

Zmola *et al.* (*9*) measured two-phase densities of an air-water system over a wide geometrical range of flow paths. These results checked with the data of Behringer in the high-volume fraction range. A radial parabolic distribution of the vapor-volume fraction was observed in the various geometries. Schwarz (*10*) investigated the density and relative velocities of the water and steam phases in vertical and horizontal boiler tubes (2.36 I.D.). He also showed a radial parabolic distribution of the vapor-volume fraction.

Up to the present no data have been published showing the liquid holdup variation due to a sudden change in the flow area resulting from either an expansion or contraction. Utilizing presently available information on the two-phase flow, one could not make a firm prediction as to whether a change in relative velocities of the two-phase mixture will occur with a change of flow area, let alone estimate the magnitude of the change. Therefore an experimental investigation was undertaken to explore further the factors which affect the relative velocity of the two phases and to attempt to provide adequate information on the effect of changes of flow area on the liquid holdup.

THEORY

The density of a two-phase mixture is given by

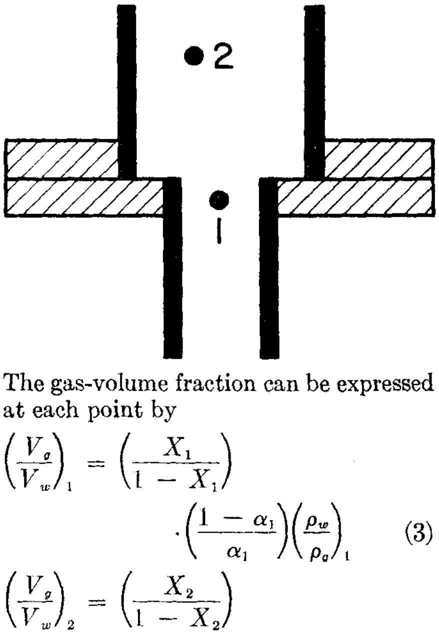
$$\rho_m = (1 - \alpha)\rho_w + \alpha\rho_g \qquad (1)$$

The relationship of the gas-volume fraction with the relative velocity of the two phases and the gas-weight fraction can be shown through the continuity equation and is

$$\frac{V_g}{V_w} = \frac{X}{1 - X}\,\frac{1 - \alpha}{\alpha}\,\frac{\rho_w}{\rho_g} \qquad (2)$$

As mentioned previously, experimental evidence indicates that the relative velocity of the two phases is a function of both circulation rate and the quality. For the case of an adiabatic system, where it can be assumed that the quality is a constant, the liquid holdup will vary only if the slip ratio changes. Further, the liquid holdup should change owing to an expansion or contraction only if the slip ratio is a function of the velocity, when one assumes that the geometry effect is negligible.

A change of flow area between two points as depicted below is considered.



The gas-volume fraction can be expressed at each point by

$$\left(\frac{V_g}{V_w}\right)_1 = \left(\frac{X_1}{1 - X_1}\right) \cdot \left(\frac{1 - \alpha_1}{\alpha_1}\right)\left(\frac{\rho_w}{\rho_g}\right)_1 \qquad (3)$$

$$\left(\frac{V_g}{V_w}\right)_2 = \left(\frac{X_2}{1 - X_2}\right) \cdot \left(\frac{1 - \alpha_2}{\alpha_2}\right)\left(\frac{\rho_w}{\rho_g}\right)_2 \qquad (4)$$

Dividing Equation (3) by Equation (4) and rearranging, one gets

$$\frac{(\alpha_1)/(1 - \alpha_1)}{(\alpha_2)/(1 - \alpha_2)} = \frac{\dfrac{(X_1/1 - X_1)(\rho_w/\rho_g)_1}{(V_g/V_w)_1}}{\dfrac{(X_2/1 - X_2)(\rho_w/\rho_g)_2}{(V_g/V_w)_2}} \qquad (5)$$

For an adiabatic system where it can be shown that

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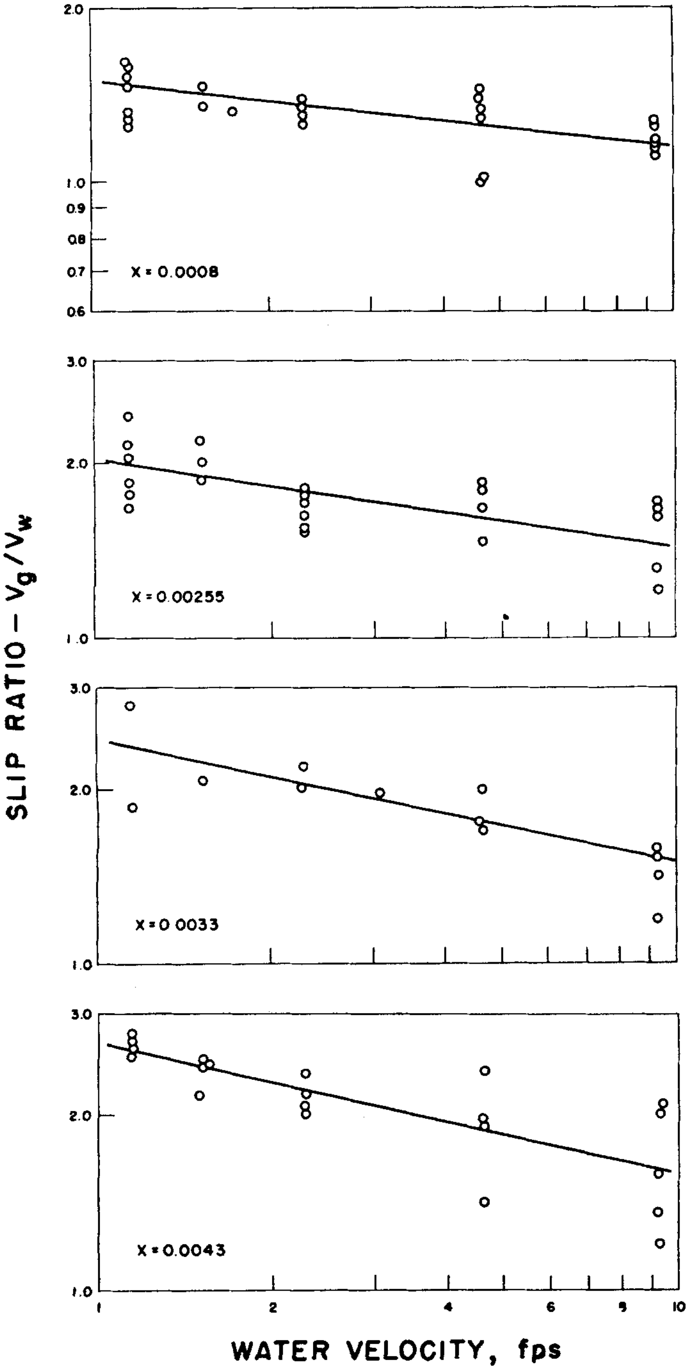

Fig. 1. Variation of slip ratio with water velocity and quality.

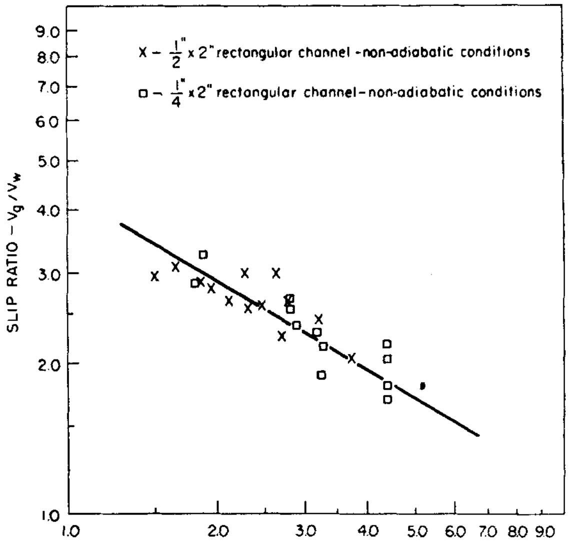

Fig. 2. Superficial water velocity V_{wo}, ft./sec.

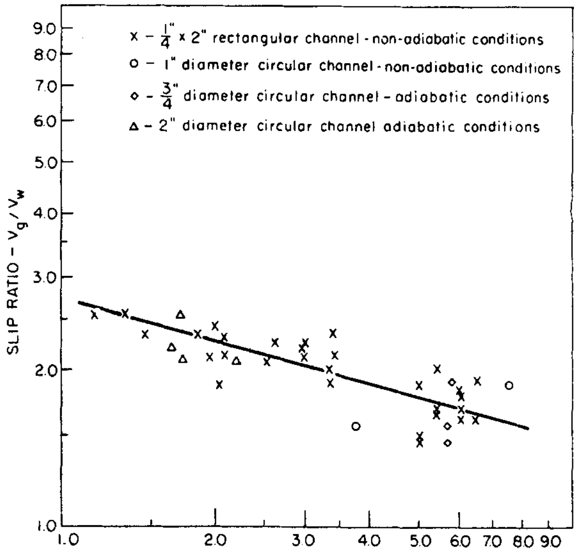

Fig. 3. Superficial water velocity V_{wo}, ft./sec.

$$X_1 = X_2$$

and

$$(\rho_w)_1 = (\rho_w)_2$$

Equation (5) becomes

$$\frac{\dfrac{\alpha_1}{1-\alpha_1}}{\dfrac{\alpha_2}{1-\alpha_2}} = \frac{\left(\dfrac{V_g}{V_w}\right)_2 (\rho_g)_2}{\left(\dfrac{V_g}{V_w}\right)_1 (\rho_g)_1} \qquad (6)$$

It may be assumed that

$$\frac{V_g}{V_w} = K V_{w_o}{}^N \qquad (7)$$

Since

$$W = V_{w_o} A \rho \qquad (8)$$

then

$$V_{w_o} = \frac{W}{A\rho} = \frac{K'''}{A} \qquad (9)$$

for the constant total flow.

Substituting Equation (9) into Equation (7) one obtains

$$\frac{V_g}{V_w} = \frac{K''}{(A)^N} \qquad (10)$$

also where

$$\frac{1}{\rho_g} = \frac{RT}{PM} \qquad (11)$$

so that

$$\rho_g = K'P \qquad (12)$$

When one substitutes Equations (10) and (12) into Equation (6) and rearranges

$$\alpha_2 = \frac{1}{\{(P_2/P_1)[(1/\alpha_1) - 1]/(A_1/A_2)^N\} + 1} \qquad (13)$$

The static pressure ratio P_2/P_1 must be included because of the large changes in the specific volumes of the gaseous phase at the lower pressures. In addition to the change of the gas-volume fraction due to the velocity effect, the gas-volume fraction will also change owing to the static pressure difference between positions. The latter effect will become negligible as the system pressure is increased ($P > 25$ lb./sq. in. abs.). The exponent N is obtained empirically.

EXPERIMENTAL APPARATUS

The experimental apparatus basically consisted of a water and air-injection system, the mixing section, the test sections, and an air-water separator. The

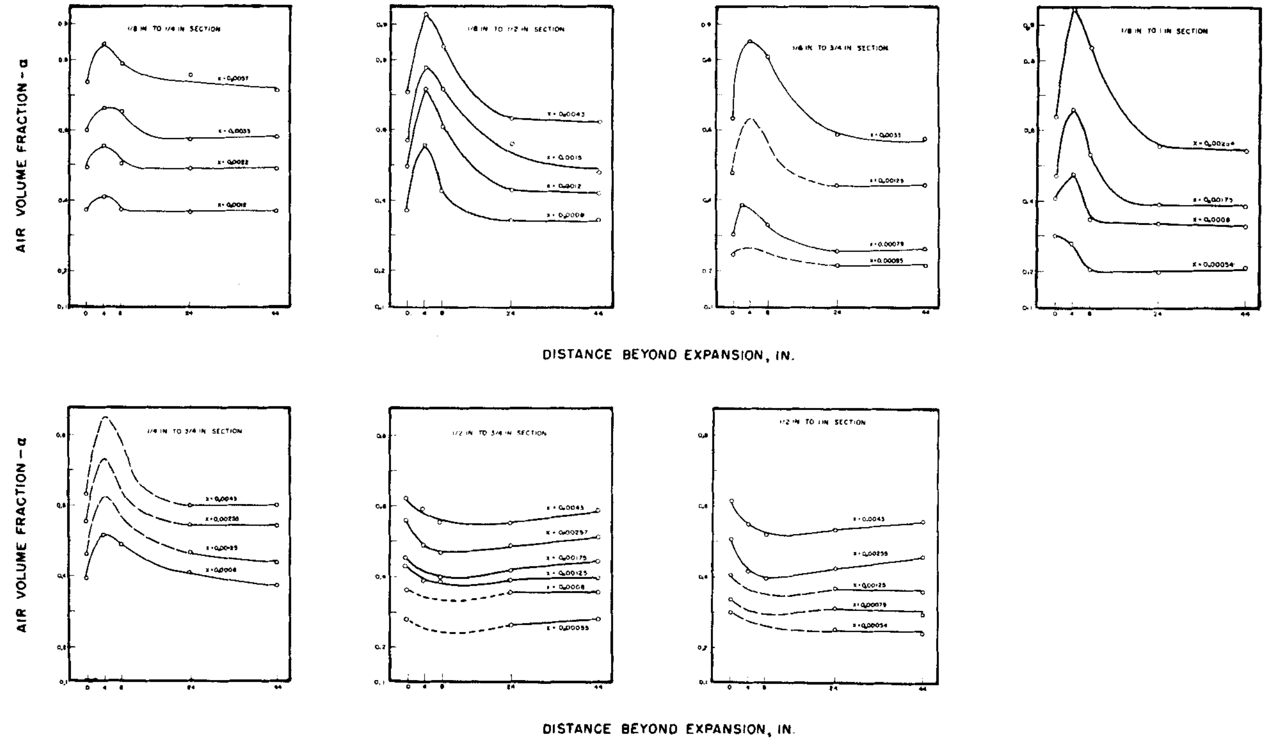

Fig. 4. Variation of air-volume fraction with length following an expansion in flow area; dashed curves represent estimated variation.

metered streams of air and water were injected into the mixer. The two-phase mixture then flowed through the test sections and into the overhead separator, where the air was liberated to the atmosphere and the water was diverted back to the make-up tank. The ranges of variables selected for study were flow-area changes, minimum 1.25:1 and a maximum of 8:1; the void-volume fractions, α, 0.2 to 0.8; mass flow rate, G, 250,000 to 2,000,000 lb./(hr.)(sq. ft.); and water velocity 1 to 10 ft./sec.

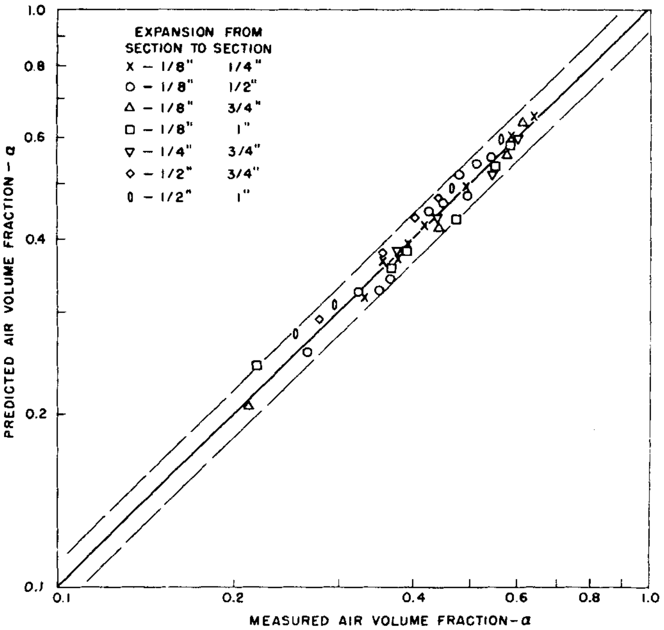

Fig. 5. Comparison of the predicted and measured air-volume fraction for a series of expansions.

Test Section

Five test sections, each 4 ft. long and 2 in. wide, with channel spacings of 1/8, 1/4, 1/2, 3/4, and 1 in., respectively, were constructed from Lucite. Lucite was used to allow visual observation and photographic studies. The sections could be interchanged to obtain the desired geometrical combinations.

Density Measurement

The equipment used for measuring the density of the two-phase mixture consisted of a 0.085-mev. thulium source, a DuMont photomultiplier tube with a sodium iodide thallium-activated scintillation crystal, a linear current amplifier, and a Brown recorder (0 to 10 mv.). The gamma rays were directed through the test section to the photomultiplier tube, where the unattenuated portion of the beam produced a signal, which was amplified and transmitted to the recorder.

The source and photomultiplier tube were mounted on a carriage which could move in either a horizontal or vertical direction. The movement of the carriage was controlled by two constant-speed motors in conjunction with a series of relays and a switch box.

The gamma beam was collimated at the photomultiplier tube by a lead window, 1 in. thick. Cooling coils were placed around the photomultiplier tube to maintain a constant temperature, since both the sodium iodide crystal and the tube are sensitive to temperature changes.

Data Procurement

A series of expansion and contraction tests was made with various section combinations. Data on expansions were obtained with the following section combinations: 1/8 to 1/4 in., 1/8 to 1/2 in., 1/8 to 3/4 in., 1/8 to 1 in., 1/4 to 3/4 in., 1/2 to 3/4 in., and 1/2 to 1 in. The section combinations used for obtaining data on contractions were 1 to 3/4 in., 1 to 1/2 in., 1 to 1/4 in., 1 to 1/8 in., 3/4 to 1/4 in., 1/2 to 1/4 in., and 1/2 to 1/8 in. It was felt that the various geometric combinations investigated gave an adequate cross section of the expansion and contraction geometries possible. (All future reference to the sections will be with respect to spacing.)

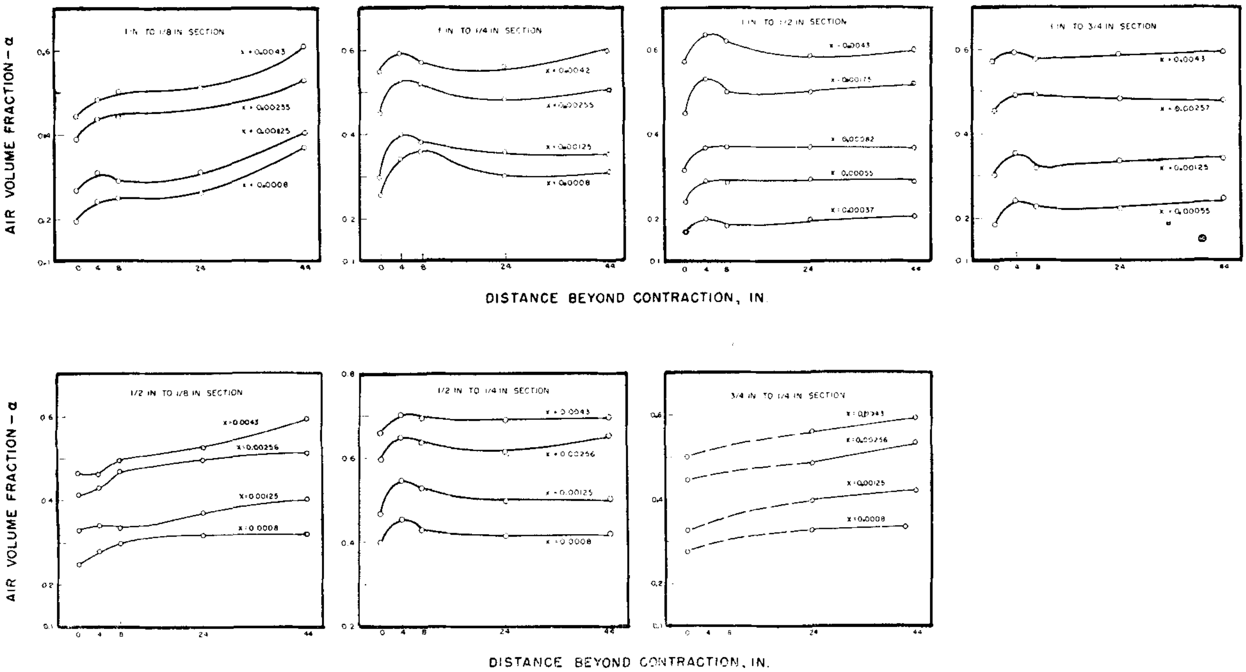

Fig. 6. Variation in air-volume fraction with length following a contraction in flow area; dashed curves represent estimated variation.

A series of runs with varying air-volume fractions was made for each geometry combination. The air-volume fraction was varied from $\alpha = 0.2$ to $\alpha = 0.75$, primarily by adjusting the flow rate of air. Owing to the relationship between specific volumes of the two phases, the quality range corresponding to the air-volume fraction range studied was very low ($X = 0.00055$ to $X = 0.0045$).

Measurements of the air-volume fraction were taken at the exit of the lower section and at positions 4, 8, 24, and 44 in. in the upper section. These positions were selected so that the transition zone could be studied (with positions at 4 and 8 in.) and the actual over-all change in the liquid holdups could be determined (with positions at 24 and 44 in.).

The air-volume fraction was measured and the data were reduced by the methods described by Petrick (*11*) and Hooker and Popper (*12*).

RELATIVE VELOCITY OF THE GASEOUS AND LIQUID PHASES

The relative velocity of the two phases was calculated for a number of data points from the measured air-volume fraction and quality and was plotted in terms of slip ratio (V_g/V_w) vs. the water velocity (based on section flow area) for constant quality parameters (Figure 1). The data show that the slip ratio is a function of both the water velocity and quality. As the velocity increases, the slip ratio decreases. Also, as the quality increases, the slip ratio increases and the velocity effect becomes more pronounced.

An interesting comparison is obtained between the air-water data of this investigation and the data obtained from boiling-water studies. The experimental loops used in these studies are described by Lottes and Petrick *et al.* (*13*). The data for steam-water mixtures in vertical channels at pressures of 150 and 600 lb./sq. in. were reduced to slip ratios in a manner analogous to that described above and are plotted as V_g/V_w vs. the superficial water velocity in Figures 2 and 3. The data shown are local values of the velocity ratio for a constant mixture quality of $X = 0.03 \pm 0.005$. The data were obtained from both adiabatic and nonadiabatic systems of widely differing geometries, which are indicated on the figures. Although there is a basic difference between the air-water and steam-water systems, there is good agreement between the two sets of data.

It can been seen that the slip ratio shows the same dependency on the superficial water velocity noted for the air-water mixtures at atmospheric pressure. As the superficial velocity increases, the slip ratio decreases. It is interesting to note that the velocity effect is greater at 150 lb./sq. in. than observed at atmospheric pressure. The fact that the effect is greater at atmospheric pressure could be due to the difference in the quality

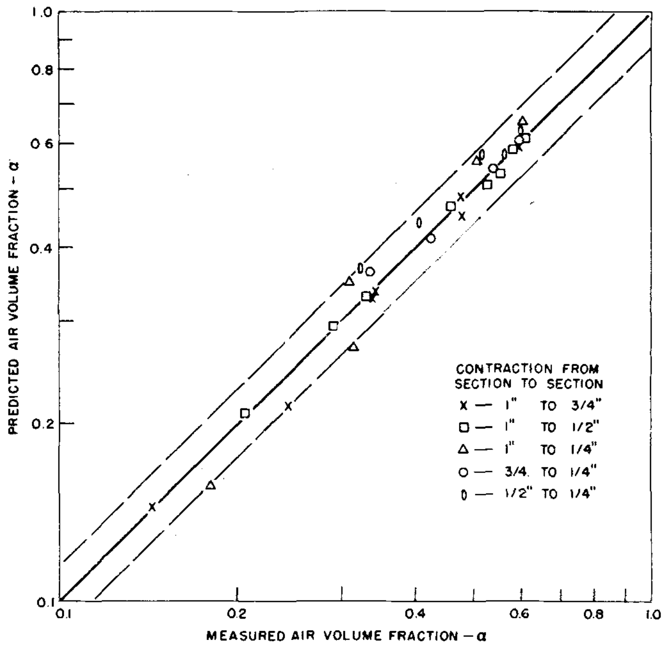

Fig. 7. Comparison of the predicted and measured air-volume fraction for a series of contractions.

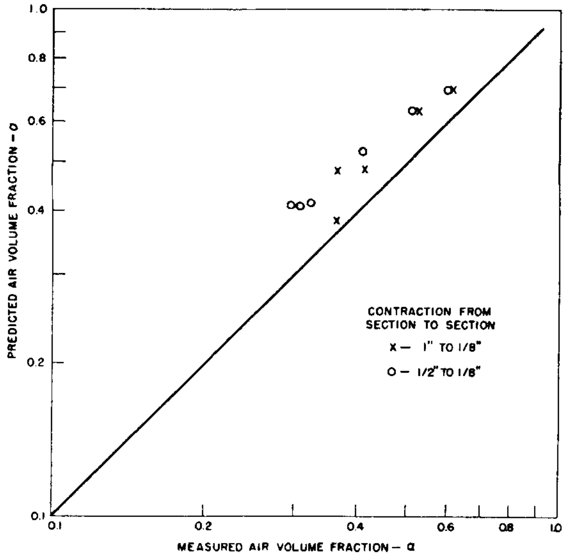

Fig. 8. Comparison of the predicted and measured air-volume fraction for a series of contractions.

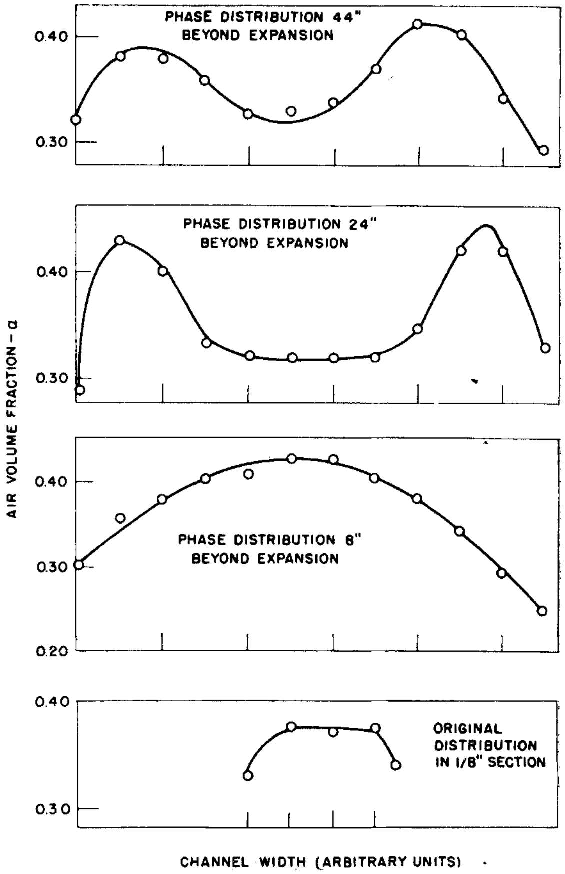

Fig. 10. Flow patterns in a ½-in. section following an expansion from a 1/8-in. section.

range. As the quality increases, the velocity effect becomes more pronounced (Figure 1). Extrapolation of the quality range of air-water data to coincide with the quality range of the steam-water data would place the values of slip ratio above the 150 lb./sq. in. data.

The decrease of the velocity effect with pressure as shown in the figures may be anticipated if the buoyancy force is acknowledged as a major factor which influences the relative velocity of the two phases. The density difference between the two phases, which is a measure of the buoyancy force, decreases with pressure and becomes zero at the critical pressure. At that point, by definition, the slip ratio must be 1, and the slip ratio therefore would be expected to decrease with pressure. As the liquid velocity tends toward zero, the buoyancy force is the dominant factor affecting the relative velocity of the two phases, when one assumes that the geometry effects are negligible. As the mass velocity of the phases increases however, the interaction between the phases increases and as a result the relative effect of the buoyancy forces on the relative velocity should diminish. Therefore the pressure effect should be more pronounced in the lower mass velocity range. Such variations would cause the velocity effect on the slip ratio to diminish as the pressure increases.

Since the slip ratio changes with the fluid velocity, the liquid holdup and hence the mean void fraction must also change following an expansion or contraction as pointed out previously.

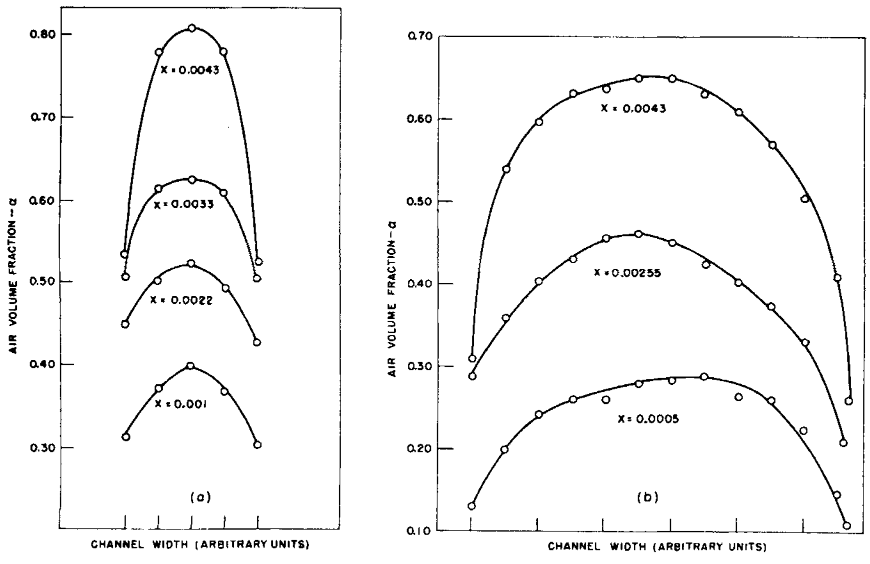

Fig. 9. Distribution of air in water in a 1/8-in. section (a) and in a 1-in. section (b) for various qualities.

AIR-VOLUME FRACTION CHANGES: EXPANSION OF FLOW AREA

The air-volume fractions are plotted as a function of the test-section length in Figure 4 for a series of expansions. It is interesting to note the variation of the void-volume fraction in the transition zone immediately following the expansion. The erratic behavior of the void-volume fraction in the transition zone is a function of the fluid velocity, mixture quality, and enlargement of the flow area. The sharp increase in void fraction shown in Figure 4 for certain conditions is due to the formation of a jet over the first few inches past the expansion and creation of air pockets. The jet dissipates into a very turbulent transition-flow region, after which the regular flow pattern is established. The severity of the jet action increases with increasing mixture quality, fluid velocity, and area enlargement.

Such severe transition zones were not found for expansions of small area enlargement with low fluid velocities and mixture quality. Under such conditions the void-volume fraction dropped immediately past the expansion and did not rise.

The data for the over-all change in the mean void fraction were correlated by Equation (13), and the exponent N was taken to be 0.2. In the correlation of the data the variation of the velocity effect with quality was neglected, since the quality range studied was not very wide and the change in the slope of the quality parameter in the plot of V_g/V_w vs. V_{w_0} was not great. Over different parameter ranges such an omission might not be tolerable. A comparison between the experimental data and the empirical fit is shown in Figure 5. The maximum deviation is $\pm$ 10%, and the average deviation 5%.

AIR-VOLUME FRACTION CHANGES: CONTRACTION IN FLOW AREA

The variation of air-volume fraction with test-section length, for the series of contractions studied, is plotted in Figure 6. In general the transition zone appears to be very short and less well defined than in the case of expansion of flow area. The air-volume fraction increases to a final value in the first few inches past the contraction.

The apparent instability of the air-volume fraction in the riser section for contractions of 1 to $\frac{1}{8}$ in. and $\frac{1}{2}$ to $\frac{1}{8}$ in., as shown in Figure 6, was due to the large static-pressure changes in the $\frac{1}{8}$-in. section which resulted from the excessive two-phase pressure drops. As the static pressure dropped along the riser length, the specific volume of the air changed markedly which, in turn, affected the air-volume fraction. Under some conditions the static pressure dropped from 30 to 15 lb./sq. in. abs. across the $\frac{1}{8}$-in. section. Such a drop would approximately double the air-volume fraction if a change in the relative velocities of the two phases did not occur. For the other series of contractions the static-pressure change across the riser section was not severe, and therefore the air-volume fraction stabilized fairly rapidly.

The change in the air-volume fraction was again calculated with Equation (13) by the use of the same value of 0.2 for N as for the expansions. The results are compared with the data in Figure 7. Again, as for the expansions, the data is adequately represented by Equation (13), with a maximum deviation of $\pm$15% and an average of 7%.

The maximum deviation occurred for contractions of from 1 to $\frac{1}{8}$ in. and from $\frac{1}{2}$ to $\frac{1}{8}$ in., as shown in Figure 8. This can be attributed to the large static-pressure changes in the $\frac{1}{8}$-in. section. Error in the predicted value of the air-volume fraction was introduced from inaccurate static-pressure readings obtained from the gauges. The flow pattern in the $\frac{1}{8}$-in. riser section was a collapsing annular type, which resulted in very severe static-pressure fluctuations. Since the magnitude of the fluctuations was, in most cases, beyond the range of the gauges, it was very difficult to obtain a mean static-pressure reading; this in turn was reflected in the predicted air-volume fraction change through the static-pressure ratio P_2/P_1. [See Equation (13).]

PHASE DISTRIBUTIONS

The extensive air-volume fraction data (obtained with the traversing technique) shows a parabolic type of distribution of the air in the liquid. Generally, the average to maximum ratio of the air-volume fraction ($\alpha_{avg}/\alpha_{max}$) which characterizes the distribution varied with the quality of the mixture and the channel spacing. Figure 9 shows typical air-liquid distributions in $\frac{1}{2}$- and 1-in. sections at various qualities. In general the distributions are symmetrical with respect to the vertical axis.

Some interesting air-volume fraction-distribution-pattern changes for expansion occurred in the riser section. A typical example is shown in Figure 10. A distribution pattern of the double annular type was formed at the midpoint of the riser. This phenomenon was observed in all riser sections at low qualities or low air-volume fractions ($\alpha > 0.4$). Except for rare instances, this type of flow pattern was not observed in sections with normal flow and no expansion of flow area. The exceptions may be due to the jet flow pattern which was shown to occur at the expansion.

CONCLUSIONS

The void-volume fraction, and hence slip ratio, of an air-water mixture changes following an expansion or contraction of flow area. The change is due to the variation of the relative velocity of the air and water phase with the fluid velocity. The relative velocity of the two phases is also a function of the mixture quality. The magnitude of the change at atmospheric pressure can be estimated by the following semitheoretical equation:

$$\alpha_2 = \frac{1}{\{(P_2/P_1)[(1/\alpha_1) - 1]/(A_1/A_2)^{0.2}\} + 1}$$

The length of the transition zone following an expansion or contraction is a function of the mixture quality, mass flow rate, and the flow area change. The transition zone following a contraction is not so pronounced as for an expansion. The distribution of the air phase in the water phase was parabolic in nature, but the ratio of maximum void-volume fraction to the average void-volume fraction, which characterizes the distribution, varied at random.

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NOTATION

- α = void-volume fraction (air, vapor, gas, etc.)
- A = cross-sectional flow area
- G = mass flow rate, lb./(hr.) (sq. ft.)
- V = fluid velocity, ft./sec.
- ρ = fluid density, lb./cu. ft.
- W = flow rate, lb./hr.
- X = mixture quality, ratio of mass flow of gas to total mass flow rate of both phases
- N = exponent in Equation (13)
- P = pressure, lb./sq. in. abs.
- T = absolute temperature, °R.
- M = molecular weight
- L = length, ft.

Subscripts

- w = liquid phase
- g = gaseous phase
- w_0 = liquid phase flowing alone in conduit
- m = two-phase mixture
- 1, 2 = position

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